ONCE AGAIN ON THE EQILIBRIUM STABILITY OF A MAN

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Resume: The given paper studies conditions of the equilibrium stability of a cylinder – shaped homogeneous body. Thus, conditions of the equilibrium stability of different proportion upright standing men were studied. A new method allowing determination of a man's common centre of gravity was developed. An optimal bending forward angle at which the degree of the equilibrium stability of an upright standing man reaches its maximum value was determined. Hence, the appropriate conclusions were drawn.

Key words: equilibrium stability; common center of gravity; optimal bending; critical angles.

PREAMBLE

The equilibrium of a body and its maintenance (stability) has a definitive significance in all fields of men's vital activities including sports. Let's examine conditions of the equilibrium stability by the example of a cylindriform homogeneous body (Fig.1). Its center of gravity (O) coincides with the mechanical center and the degree of its equilibrium stability comes to a maximum value of expression when it is at a strictly vertical position (Fig.1a).

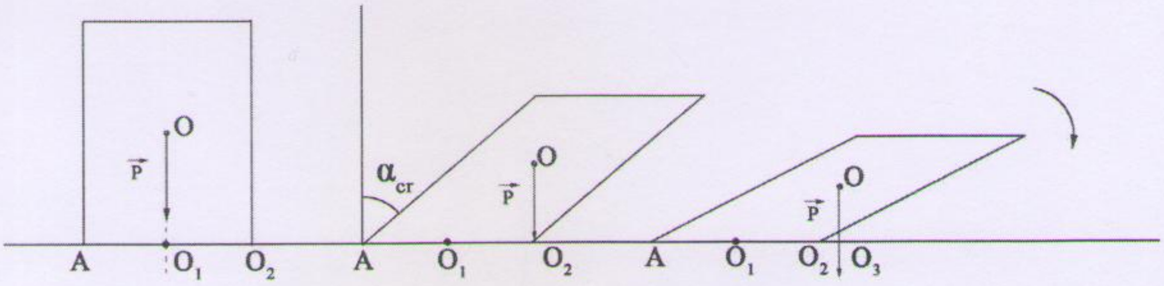


Fig.1
Process of gradual bending of a cylindriform body having fixed base

At such position the vertical projection of the center of gravity of the cylindriform body onto the base area, coincides with the mechanical center of the base (O1) and the moment of gravity of the

cylinder $M = |\vec{P}| \cdot \frac{d}{2}$, as regards the center of rotation (O_2), has maximum value of significance and serves as a restricting function (i.e. is a restricting moment), thus ensuring maximum degree of the equilibrium stability (|P|- gravity value, d=|AO_2)- base diameter, $\frac{d}{2}$ -arm of gravity).

With the cylinder's gradual deviation (on conditions that the base is fixed) both the arm of gravity and the value of the restricting moment diminish resulting in the decrease in the cylinder's equilibrium stability degree.

At the position given in Fig.1b, the cylinder's vertical projection of gravity onto the base area coincides with the center of rotation and the restricting moment (a), therefore, the equilibrium stability degree comes practically to zero (critical angle of deviation - α_{cr}). At the subsequent deviation of the cylinder, the gravity moment changes its sign and acquires the function of a rotational moment and the body begins to topple over (Fig.1c).

Considering the said above, one may conclude: the degree of the body's equilibrium stability becomes maximum when the vertical projection of its center of gravity onto the base area coincides with the mechanical center of the base; i.e. while the vertical projection of the body's gravity is within the base area, the body maintains its equilibrium position but when the given projection leaves the area, the body fails equilibrium and toppling over begins.

The dependence of the critical angle values (α_{cr}) on the cylindriform body ($\frac{H}{d}$), where (H) is the height of the cylinder and (d) is the base diameter.

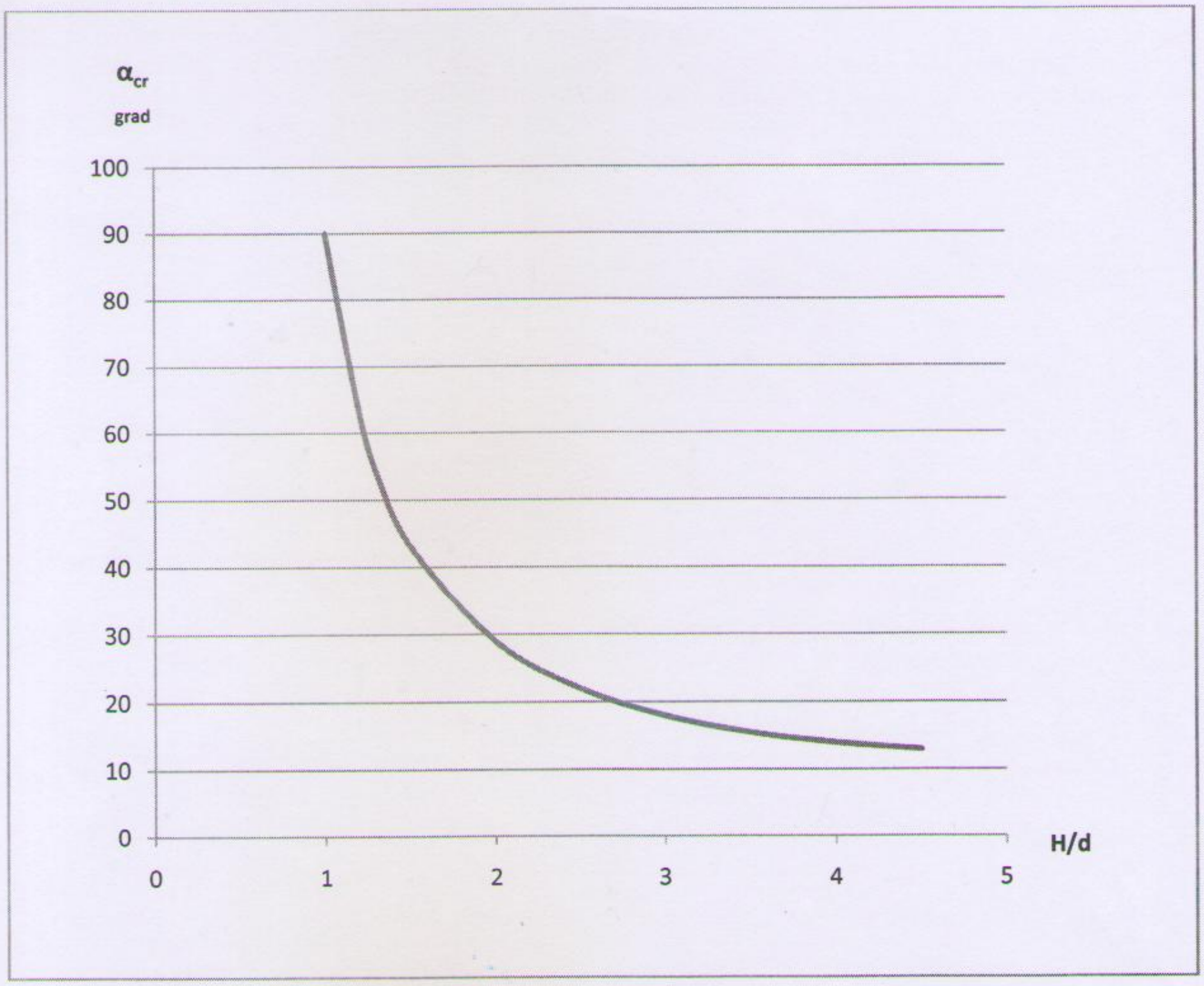
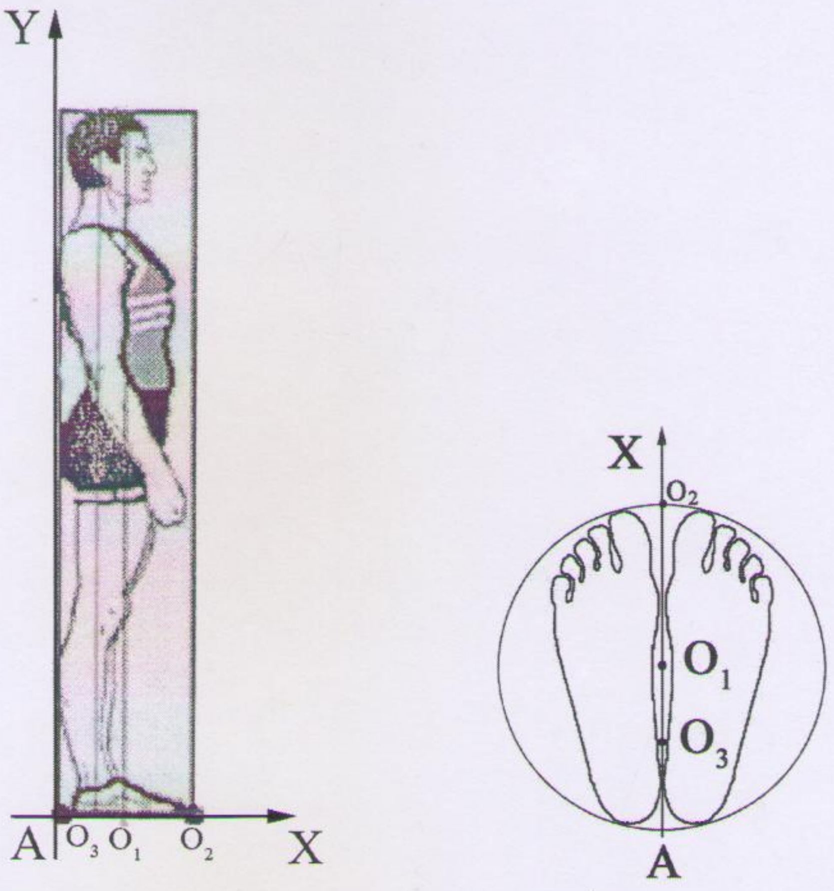


Fig.2

Dependence of the critical angle value of the cylindriform body on its geometry $\alpha_{cr} = f\left(\frac{H}{d}\right)$

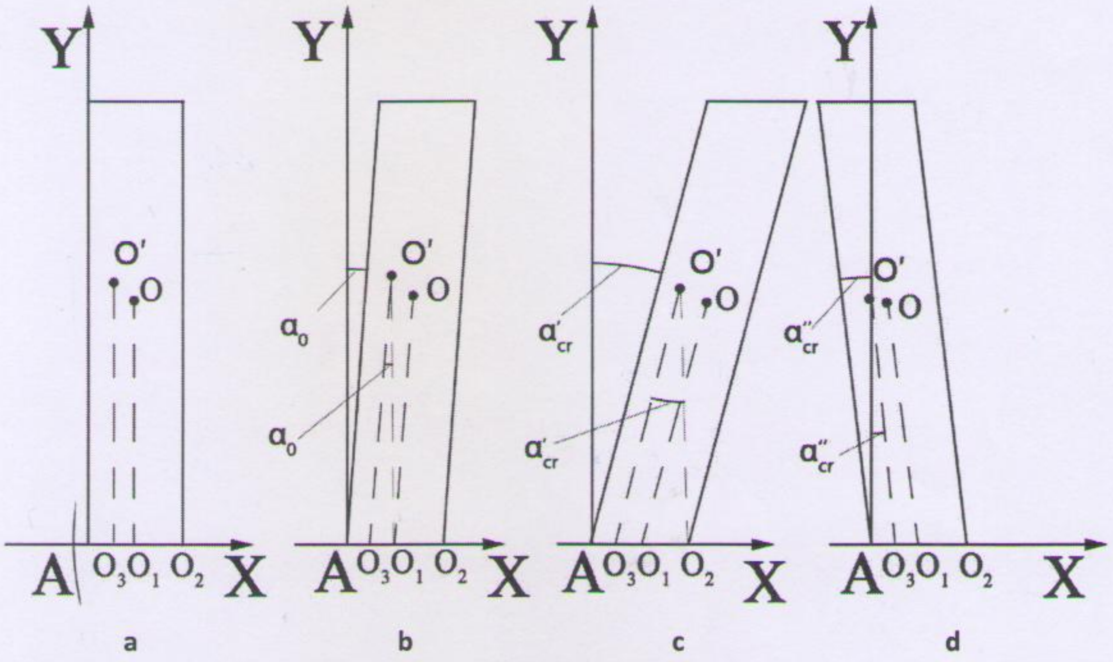
MAIN PART

Fig.4a clearly demonstrates that If an upright standing man is placed into an imaginary (virtual) cylinder (Fig.3a), the body weight will be distributed unevenly inside the cylinder and the general center of gravity (GCG) of the man will not coincide with the cylinder's mechanical center. The GCG (O') of a man and the mechanical center of the cylinder (O) are displaced considerably against each other due to the specificity of the men's anatomy.



a b

Fig.3
a-An upright standing man placed into an imaginary (virtual) cylinder.
b-Feet position on the bearing area



a b c d

Fig.4

a – Position of the general gravity center (O') of an upright standing man

b – Optimal position of a man when the equilibrium stability degree is a maximum and the optimal bending forward angle (α_o) correlates with it.

c - Critical situation occurs when a man bends forward and the equilibrium stability degree equals zero and toppling over of the body takes place; critical angle of bending forward $\alpha'_{cr.}$.

d - Critical situation occurs when a man bends backward; critical angle $\alpha'_{cr.}$. Body topples over backward.

Feet position of a man on the cylinder base is given in Fig.3b, where point O_1 is a projection of the cylinder's mechanical center (O) (Fig.4a) coinciding with the mechanical center of the base area (O_1), whereas O_3 point is a projection of the man's GCG onto the same area; since the GCG projections of the man (O') and the cylinder's mechanical center on the base area are reciprocally displaced, the equilibrium stability degree cannot have its maximum value; to succeed, the upright standing man should bend forward (without tearing the feet away from the foothold) at an optimal angle (α_o) so that the vertical projection of the man's GCG (of O' point) onto the base area gets coincided with its mechanical center (O_1). The given situation is presented in Fig.4b and enables to determine the optimal angle of bending [1]:

$$\alpha_0 = \arcsin\left(\frac{|O_3O_1|}{|O_3O'|}\right). \quad (1)$$

In much the same way critical angles of bending forward α'_{cr} and backward α''_{cr} can be determined by the following formulas (Fig.4c and 4d):

$$\alpha'_{cr} = \arcsin\left(\frac{|O_3O_2|}{|O_3O'|}\right), \quad (2)$$

$$\alpha''_{cr} = \arcsin\left(\frac{|AO_3|}{|O_3O'|}\right). \quad (3)$$

It is well known {1,2} that: $|AO_3| = |O_3O_1| = \frac{d}{4}$, $|O_3O_2| = \frac{3}{4}d$, $|O_2O_1| = \frac{d}{2}$, and $|O_3O'| = h$

Taking the aforesaid into consideration, formulas (1), (2) and (3) will become:

$$\alpha_0 = \arcsin\left(\frac{d}{4h}\right), \quad (1')$$

$$\alpha'_{cr} = \arcsin\left(\frac{3d}{4h}\right), \quad (2')$$

$$\alpha''_{cr} = \arcsin\left(\frac{d}{4h}\right), \quad (3')$$

Where d=IAO_2I is the diameter of the base area (length of the man's foot), h – the height of GCG position from the same area.

In order to determine the height of the GCG position of the upright standing man (h=IO_3O'I) by means of the man' height (H), we make use of Phidias Number (or "Fibonacci Number" and "Golden Ratio" equivalently). Phidias Number (?=1.6118) determines the ratio between the man's height H and the height of the man's umbilicus (h'):

$$\varphi = \frac{H}{h'} \tag{4}$$

Also, it is well known that the GCG of an upright standing man is below the umbilicus by 0.05, and as the women are concerned, it is by 0.1·H, since their umbilicus is lower than the one of the men by 0.05·H. Taking into consideration both the said above and formula (4) we derive for men:

$$h = H \cdot \left(\frac{1}{\varphi} - 0{,}05\right), \tag{5}$$

$$h = 0{,}5680 \cdot H;$$

and for women:

$$h = H \cdot \left(\frac{1}{\varphi} - 0{,}1\right), \tag{5'}$$

$$h = 0{,}5180 \cdot H.$$

If formula (5) is taken into consideration, formulas (1'), (2') and (3') will become:

$$\alpha_0 = \arcsin\left(\frac{1}{4\left(\frac{1}{\varphi} - 0{,}05\right)} \cdot \frac{d}{H}\right), \tag{6}$$

$$\alpha'_{cr} = \arcsin\left(\frac{3}{4\left(\frac{1}{\varphi} - 0{,}05\right)} \cdot \frac{d}{H}\right), \tag{7}$$

$$\alpha''_{cr} = \arcsin\left(\frac{1}{4\left(\frac{1}{\varphi} - 0{,}05\right)} \cdot \frac{d}{H}\right). \tag{8}$$

Taking into consideration that $\varphi = 1{,}618$, we receive:

$$\alpha_0 = \arcsin\left(0{,}44 \cdot \frac{d}{H}\right), \tag{6'}$$

$$\alpha'_{cr} = \arcsin\left(1{,}32 \cdot \frac{d}{H}\right), \tag{7'}$$

$$\alpha''_{cr} = \arcsin\left(0{,}44 \cdot \frac{d}{H}\right). \tag{8'}$$

In case of women and taking into consideration (5'), formulas (6'), (7') and (8')will become:

$$\alpha_0 = \arcsin\left(0{,}48 \cdot \frac{d}{H}\right), \tag{6''}$$

$$\alpha'_{cr} = \arcsin\left(1{,}44 \cdot \frac{d}{H}\right), \tag{7''}$$

$$\alpha''_{cr} = \arcsin\left(0{,}48 \cdot \frac{d}{H}\right). \tag{8''}$$

It is generally known that the Phidias Number is the most harmonic ratio in the nature of things; sculptors, painters and architects of the ancient Egypt, of the antique epoch and Renaissance used it. Leonardo da Vinci used the Number in his famous work on "the human figure proportion". (Fig.5).

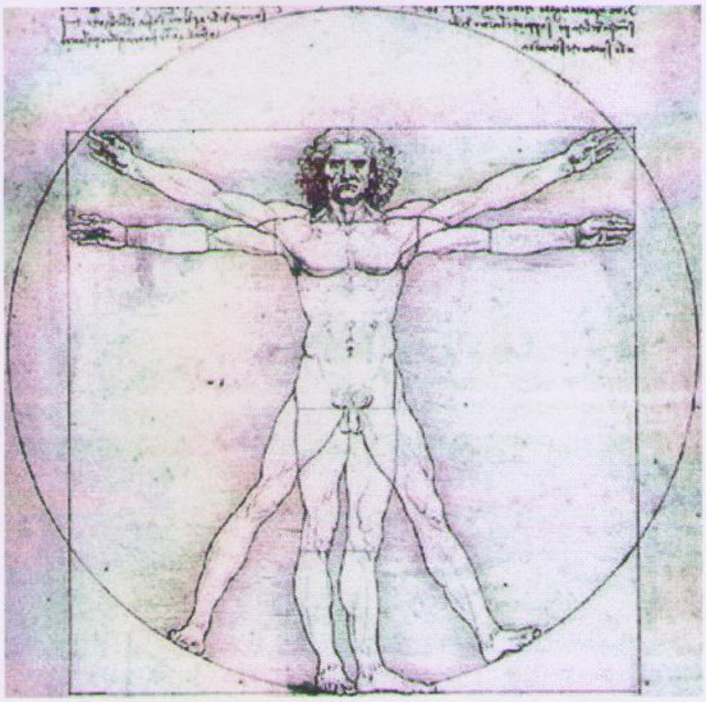


Fig. 5

Leonardo da Vinci. "Proportion of the human figure".

For a man of the Da Vinci proportion (Fig.5), both optimal α_0 and critical α_{cr}', α_{cr}'' have the following value:

$$\alpha_0 = 3{,}7^0,\ \alpha_{cr}' = 11{,}15^0,\ \alpha_{cr}'' = 3{,}7^0.$$

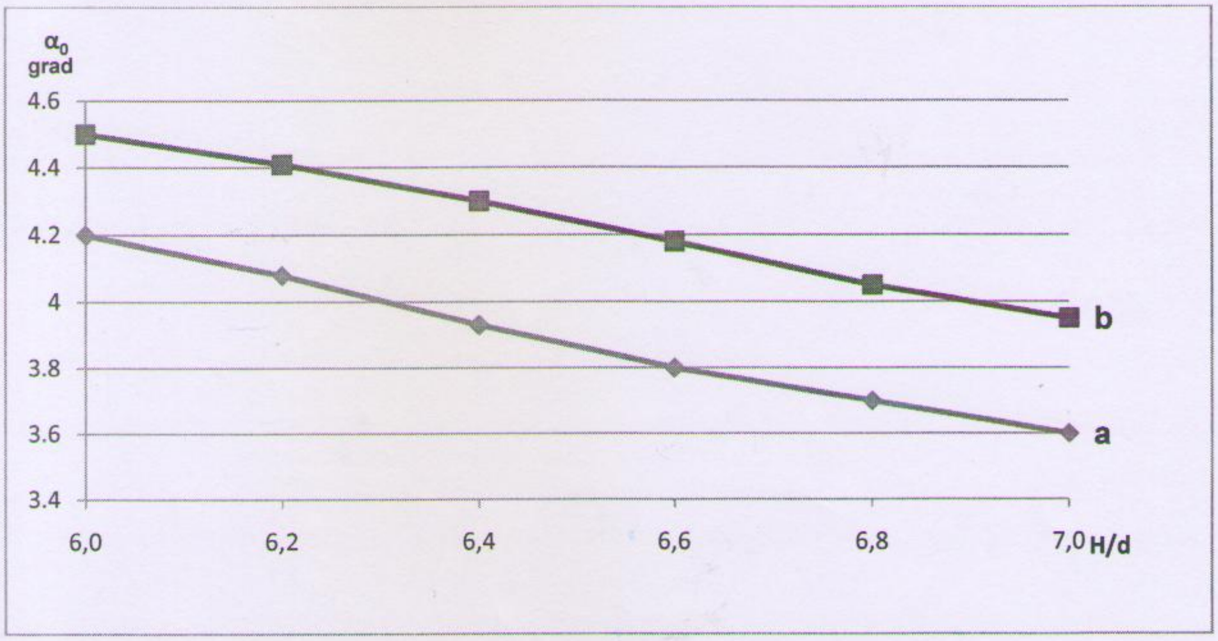


Fig.6

Dependence of the optimal angle value (α_0) on the man's proportion $\left(\frac{H}{d}\right)$.

a-for men; b-for women (The average value $\left(\frac{H}{d}\right)$ for a proportional man varies within 6÷7)

Comparing the given diagrams (Fig.6) one can conclude that the degree of the equilibrium stability of women is higher than of the men of similar proportion caused by a lower position of their (women) GCG. Similarly, a formula determining skier's optimal angle of bending forward can be drawn:

$$\alpha_0 = \arcsin\left[\frac{1}{H}\left(1{,}32 \cdot d + \frac{n \cdot l}{0{,}568}\right)\right]$$

Where d-length of foot, l-length of ski, H-height of skier, n-coefficient determining the part of the ski length made up by the distance from the tip of foot to the geometric center of the part of the ski's bearing area.

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